Braids, Links, and Applications

Hybrid Conference
IMSA Miami, 15-18 November 2021

Program

Monday 15 November 2021

10:00 am – 11:00 am	Sofia Lambropoulou
11:15 am – 12:15 pm	Mario Salvetti
2:00 pm – 3:00 pm	Anatoly Libgober
3:15 pm – 4:15 pm	Alex Suciu
4:30 pm – 5:30 pm	Artan Sheshmani

Tuesday 16 November 2021

9:15 am – 10:15 am	Moshe Cohen
10:20 am – 11:20 am	Rick Miranda
11:30 pm – 12:30 pm	Sergei Gukov
1:50 pm – 2:50 pm	Giovanni Paolini
3:00 pm – 4:00 pm	Ignat Soroko

Wednesday 17 November 2021

10:00 am – 11:00 am	Christine Ruey Shan Lee
11:30 am – 12:30 pm	Sergei Gukov
2:00 pm – 3:00 pm	Nikolai Saveliev
3:15 pm – 4:15 pm	Iva Halacheva
4:30 pm – 5:30 pm	Nancy Scherich

Thursday 18 November 2021

10:00 am – 11:00 am	Ken Baker
11:15am – 12:15 pm	Josef Svoboda
2:00 pm – 3:00 pm	Dani Pallavi
3:15 pm – 4:15 pm	Kyoung-Seog Lee

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Abstracts

Bounds on Morse-Novikov Numbers of 3-Manifolds
Ken Baker

The Morse-Novikov number counts the minimum number of critical points among circle valued Morse functions in a homotopy class. Prompted by Pajitnov, we demonstrate "hands-on" bounds on Morse-Novikov numbers of 3-manifolds in terms of the Morse-Novikov number of the trivial class. In doing so we are led to a curious function on cohomology with a toy model concerning links in the thickened torus. This is joint work with Fabiola Manjarrez-Gutierrez.

The Average Genus of a 2-Bridge Knot Grows Linearly with Respect to Crossing Number
Moshe Cohen

Dunfield et al provide experimental data to suggest that the Seifert genus of a knot grows linearly with respect to crossing number. We prove this holds among 2-bridge knots using Chebyshev billiard table diagrams developed by Koseleff and Pecker. This work builds on results by the first author with Krishnan and Even-Zohar and Krishnan on a random model using these diagrams. This work also uses and improves upon results by the author demonstrating a lower bound for the average genus among a weighted collection of 2-bridge knots.

This is joint work with Adam Lowrance.

From Turaev to Rokhlin via VOA Characters and Curve Counting I & II
Sergei Gukov

These lectures will present a board survey of recent work on new q-series invariants of 3-manifolds labeled by Spin-C structures. While the original motivation for studying these invariants is rooted in topology, they exhibit a number of unexpected properties and connections to other areas of mathematics, e.g. turn out to be characters of logarithmic vertex algebras. The integer coefficients of these q-series invariants can be understood as the answer to a certain enumerative problem, and when q tends to special values these invariants relate to other invariants of 3-manifolds labeled by Spin and Spin-C structures.

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Welded Tangles and a Topological Interpretation of the Kashiwara-Vergne Group
Iva Halacheva

Welded or w-tangles are a higher dimensional analogue of classical tangles, which admit a yet further generalization to welded foams, or w-trivalent graphs, a class of knotted tubes in 4-dimensional space. Welded foams can be presented algebraically as a circuit algebra.

Together with Dancso and Robertson we show that their automorphisms can be realized in Lie theory as the Kashiwara-Vergne group, which plays a key role in the setting of the Baker-Campbell-Hausdorff series. In the process, we use a result of Bar-Natan and Dancso which identifies homomorphic expansions for welded foams, a class of powerful knot invariants, with solutions to the Kashiwara-Vergne equations.

Braidings and Braid Equivalences
Sofia Lambropoulou

We present braiding algorithms and derive braid equivalences for knots and links in various topological settings (classical, in 3-manifolds, virtual, singular, long knots), focusing in the end in the recent theory of knotoids and braidoids.

Crossing Numbers of Whitehead Doubles
Christine Ruey Shan Lee

Determining the minimal crossing number of a knot is typically a difficult problem. In particular, not much is known about the behavior of crossing numbers under the operation of taking Whitehead doubles. In this talk, we will discuss how the colored Jones polynomial can be used to study the crossing numbers of Whitehead doubles. We use the connection between the asymptotics of the degrees of the colored Jones polynomial and crossing numbers to determine the minimum crossing numbers of an infinitely family of satellite knots. This is joint work with Efstratia Kalfagianni.

Hilbert Schemes of Plane Curve Singularities and Matrix Factorizations
Kyoung-Seog Lee

Plane curve singularities have provided bridges between algebraic geometry and low dimensional topology. For example, the HOMFLY-PT polynomial of an algebraic link can be expressed in terms of Hilbert schemes of the plane curve singularity thanks to the works of Oblomkov-Shende and Maulik. On the other hand, there have been lots of interests in mirror symmetry of hypersurface singularities these days and plane curve singularities again have provided natural testing grounds for mirror symmetry conjecture. In this talk, we will discuss the relation between Hilbert schemes of plane curve singularities, certain topological data of some algebraic links, and matrix factorizations, stability conditions on them.

Pencils on Algebraic Surfaces with Prescribed Classes of Irreducible Components
Anatoly Libgober

We describe a refinement of the bounds on the number of reducible fibers in a pencil of curves on a smooth projective surface assuming that irreducible components of reducible members belong to a fixed subset of Neron-Severi group. The results give a substantial generalization of results by the speaker on free quotients of the fundamental groups of the complements to arrangements of lines. Joint work with J.I. Cogolludo.

Moduli Spaces for Rational Elliptic Surfaces (of index 1 and 2)
Rick Miranda

Elliptic surfaces form an important class of surfaces both from the theoretical perspective (appearing in the classification of surfaces) and the practical perspective (they are fascinating to study, individually and as a class, and are amenable to many particular computations). Elliptic surfaces that are also rational are a special sub-class.

The first example is to take a general pencil of plane cubics (with 9 base points) and blow up the base points to obtain an elliptic fibration; these are so-called Jacobian surfaces, since they have a section (the final exceptional curve of the sequence of blowups). Moduli spaces for rational elliptic surfaces with a section were constructed by the speaker, and further studied by Heckman and Looijenga. In general, there may not be a section, but a similar description is possible: all rational elliptic surfaces are obtained by taking a pencil of curves of degree 3k with 9 base points, each of multiplicity k.

There will always be the k-fold cubic curve through the 9 points as a member, and the resulting blowup produces a rational elliptic surface with a multiple fiber of multiplicity m (called the index of the fibration). A. Zanardini has recently computed the GIT stability of such pencils for m=2; in joint work with her we have constructed a moduli space for them via toric constructions. I will try to tell this story in this lecture.

Subgroups of Right-Angled Coxeter Groups via Stallings-like Techniques
Dani Pallavi

Stallings folds are a tool that has had a tremendous impact in the study of subgroups of free groups. For instance, they can be deployed to easily determine whether a subgroup is normal, to find a basis, or to determine its index. After a brief introduction, I will talk about joint work with Ivan Levcovitz, in which we develop an analogue of such folds for the setting of right-angled Coxeter groups.

I will describe several applications, including a recent new construction of non-quasiconvex subgroups of hyperbolic groups.

The $K(\pi, 1)$ Conjecture II
Giovanni Paolini

We introduce to the proof of the classical $K(\pi,1)$ conjecture, recently given in the case of affine Artin groups (G. Paolini, M. Salvetti, "Proof of the $K(\pi, 1)$ conjecture for affine Artin groups", Inven. Math, {\bf 224}, 2 (2021)").

The $K(\pi, 1)$ Conjecture I
Mario Salvetti

We introduce to the proof of the classical $K(\pi,1)$ conjecture, recently given in the case of affine Artin groups (G. Paolini, M. Salvetti, "Proof of the $K(\pi, 1)$ conjecture for affine Artin groups", Inven. Math, {\bf 224}, 2 (2021)").

On the Deleted Squares of Lens Spaces
Nikolai Saveliev

The configuration space F2(M) of ordered pairs of distinct points in a manifold M, also known as the deleted square of M, is not a homotopy invariant of M: Longoni and Salvatore produced examples of homotopy equivalent lens spaces M and N of dimension three for which F2(M) and F2(N) are not homotopy equivalent. We study the natural question whether two arbitrary 3-dimensional lens spaces M and N must be homeomorphic in order for F2(M) and F2(N) to be homotopy equivalent. Among our tools are the Cheeger–Simons differential characters of deleted squares and the Massey products of their universal covers. This is a joint work with Kyle Evans-Lee.

Representations of the Classical, Virtual, and Welded Braid Groups
Nancy Scherich

The virtual and welded braid groups are generalizations of the classical braid group by adding new kinds of crossings and relations. We will discuss constructions of classical braid group representations that extend to representations of the virtual and welded braid groups.

Global Shifted Potentials for Moduli Stack of Sheaves Over Calabi-Yau 4 Folds
Artan Sheshmani

We report on series of joint works with Borisov, Katzarkov and Yau on construction of a globally defined shifted potential functionals over moduli stack of stable sheaves on Calabi-Yau 4 folds. Firstly, we discuss globally defined shifted symplectic structures on DG Quot schemes of sheaves over DG manifolds. The theory of dg Quot schemes is developed so that it becomes a homotopy site, and the corresponding infinity category of stacks is equivalent to the infinity category of stacks, as constructed by Toen and Vezzosi, on the site of dg algebras whose cohomologies have finitely many generators in each degree. Stacks represented by dg schemes are shown to be derived schemes under this correspondence.

Then we discuss generalization of our construction over DG Quot stacks, or stacky quotient of of our DG schemes. We show that any derived scheme over C equipped with a (-2)-shifted symplectic structure, and having a Hausdorff space of classical points, admits a globally defined Lagrangian distribution as a dg C-infinity manifold.

We will use this result to construct Lagrangian distributions on stable loci of derived Quot-stacks. The main tool for proving our main theorem is a strictification result for Lagrangian distribution.

Finally, we show that there are globally defined Lagrangian distributions on the stable loci of derived Quot-stacks of coherent sheaves on Calabi–Yau four-folds. Dividing by these distributions produces perfectly obstructed smooth stacks with globally defined (-1)-shifted potentials, whose derived critical loci give back the stable loci of smooth stacks of sheaves in global Darboux form. This report is based on joint papers: arXiv:1908.03021, arXiv:1908.00651, and arXiv:2007.13194

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Divergence in Coxeter Groups

Ignat Soroko

Divergence of a metric space is an interesting quasi-isometry invariant of the space which measures how geodesic rays diverge outside of a ball of radius r, as a function of r. Divergence of a finitely generated group is defined as the divergence of its Cayley graph. For symmetric spaces of non-compact type the divergence is either linear or exponential, and Gromov suggested that the same dichotomy should hold in a much larger class of non-positively curved CAT(0) spaces.

However, this turned out not to be the case and we now know that the spectrum of possible divergence functions on groups is very rich. In a joint project with Pallavi Dani, Yusra Naqvi, and Anne Thomas, we initiate the study of the divergence in the general Coxeter groups. We introduce a combinatorial invariant called the `hypergraph index', which is computable from the Coxeter graph of the group and use it to characterize when a Coxeter group has linear, quadratic or exponential divergence, and also when its divergence is bounded by a polynomial.

Braids and Line Arrangements - Old and New

Alex Suciu

Braid groups, configuration spaces, and hyperplane arrangements have been tightly intertwined for at least 60 years. I will discuss some recent advances in our understanding of fundamental groups of complements of complex line arrangements, with emphasis on several of the Lie algebras associated to them.

GPPV Invariants and the Spectrum of Singularities

Josef Svoboda

To an isolated complex surface singularity, we can assign a 3-manifold - the link of the singularity. There has been a lot of recent work on modularity properties of GPPV invariants $\hat(Z)_b$ of such 3-manifolds. I will present a new relation of these modular forms with the spectrum of the corresponding singularity in the case of Brieskorn homology spheres and links of ADE singularities. Joint work with L. Katzarkov and Kyoung-Seog Lee.