Canonical Weierstrass representation of minimal Lorentz surfaces in pseudo-Euclidean 4-space with neutral metric

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The minimal Lorentz surfaces in the pseudo-Euclidean 4-space with neutral metric $\mathbb{E}^4_2$ whose first normal space is two-dimensional and whose Gauss curvature K and normal curvature $\varkappa$ satisfy the condition $K^2 - \varkappa^2 \neq 0$ are called minimal Lorentz surfaces of general type. We prove that these surfaces admit special isothermal parameters, called canonical. The Gauss curvature K and the normal curvature $\varkappa$ of such a surface considered as functions of the canonical parameters satisfy the following system of natural PDEs:

$$\begin{array}{rcl} \sqrt[4]{\left|K^2 - \varkappa^2\right|}\, \Delta^h \ln \left|K^2 - \varkappa^2\right| & = & 8K; \\ \sqrt[4]{\left|K^2 - \varkappa^2\right|}\, \Delta^h \ln \left|\dfrac{K + \varkappa}{K - \varkappa}\right| & = & 4\varkappa; \end{array} \qquad K^2 - \varkappa^2 \neq 0. \tag{1}$$

We obtain a Weierstrass representation with respect to canonical parameters of any minimal Lorentz surface of general type and describe all these surfaces in terms of four real functions. Using the canonical Weierstrass representation we solve explicitly system (1) expressing any solution to this system by means of four real functions of one variable. Finally, we give examples of minimal Lorentz surfaces of general type in $\mathbb{E}^4_2$ parametrized by canonical parameters.

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