Coassociative fibrations.

The Euclidean space $\mathbb{R}^7$ possesses some amazing geometrical features as it is a unique higher-dimensional vector space admitting a structure akin to the vector product in $\mathbb{R}^3$. As a particular attribute, this so-called $G_2$ structure allows us to define some intriguing submanifolds.

One class of such submanifolds is known as co-associative 4-folds.

Remarkably, it can be shown that these coassociative submanifolds are volume-minimizing among all nearby submanifolds if the respective $G_2$ structure is torsion-free. In particular, this bears an analogy to special Lagrangian submanifolds of Calabi–Yau manifolds.

I will explain the role of differential forms in setting up a $G_2$ geometry in 7 dimensions and the deformation theory of coassociative submanifolds.

Then I will give informal description of different constructions of examples where the deformation family “fills” the ambient 7-manifold $M$, thereby defining a fibration of $M$ by coassociative submanifolds (with some singular fibres).

On nearly parallel $G_2$-manifolds.

A nearly parallel $G_2$ structure on a 7-manifold can be given by a 3-form $\varphi$ of special algebraic type satisfying a differential equation $d\varphi = \tau \wedge \varphi$ for a non-zero constant $\tau$. We consider nearly parallel $G_2$ structures on the Aloff–Wallach spaces and on regular Sasaki–Einstein 7-manifolds. We give a construction and explicit examples of associative 3-folds (a particular type of minimal submanifolds) in these spaces. Joint work with M. Fernández, A. Fino and V. Muñoz.

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