A Construction of Apéry Constants from Landau-Ginzberg Models

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The irrationality of the Riemann zeta function at 3 was historically proven by R. Apéry by finding a rapidly converging sequence which is consisted of two sequences in integers and rationals satisfying certain recursive relations. Nowadays it is known that this sequence is obtained from the power series expansion of the holomorphic period function of a family of K3 surfaces, and the recurrences arise from the Picard-Fuchs differential equation.

For some Fano threefolds with Picard rank 1, V. Golyshev obtained similar special values of L-functions as Apéry limit of the quantum differential equations. If one believes the mirror symmetry also preserves these arithmetic special values, there should be a “mirror” construction in the B-model side. In this talk, as an evidence I introduce constructions of geometric higher normal functions on the mirror Landau-Ginzberg models of the above Fano threefolds. Limiting values of these normal functions toward singular fibers reconstruct the Apéry constants computed in the A-model side. With Mukai’s classification of the Fano threefolds, the results for $V_{10}, V_{12}, V_{16}, V_{18}$ are shown by M. Kerr and G. Silva Jr. A partial result for the $V_{14}$ case is given by the speaker.