Moduli and Hodge Theory

Virtual Conference
IMSA Miami, 1-5 February 2021

Program

Monday 1 February 2021

9:00 am – 10:00 am	**Dan Abramovich**
10:30 am – 11:30 am	**Radu Laza**
1:30 pm – 2:30 pm	**Valery Alexeev**

Tuesday 2 February 2021

9:00 am – 10:00 am	**Michael Temkin**
10:30 am – 11:30 am	**Colleen Robles**
1:30 pm – 2:30 pm	**Mark Green**

Wednesday 3 February 2021

9:00 am – 10:00 am	**Sönke Rollenske**
10:30 am – 11:30 am	**Giancarlo Urzúa**
1:30 pm – 2:30 pm	**Julie Rana**

Thursday 4 February 2021

9:00 am – 10:00 am	**Marco Franciosi**
10:30 am – 11:30 am	**Rita Pardini**
1:30 pm – 2:30 pm	**Giulia Sacca**

Friday 5 February 2021

9:00 am – 10:00 am	**Sam Payne**
10:30 am – 11:30 am	**Paul Hacking**

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Abstracts

Semistable Reduction - A Progress Report
Dan Abramovich

How well can one resolve the singularities of a family of complex varieties? The problem was raised in 1973 by Mumford on page vii of "Toroidal Embeddings I". A precise conjecture, the semistable reduction conjecture, was made by Karu and me in our 2000 paper, where we prove weak semistable reduction. I will explain recent work with Temkin and Wlodarczyk on functorial weak semistable reduction, and discuss work of Adiprasito, Liu and Temkin which proves the semistable reduction conjecture.

Compact moduli of polarized K3 surfaces
Valery Alexeev

We prove that the natural geometrically meaningful compactification of the moduli of lattice polarized K3s corresponding to a canonical polarizing divisor, that is well-behaved under degenerations, is semi toroidal up to normalization. We also show that for ordinary polarized K3s of degree 2d the sum of rational curves in the polarization is such a well-behaved divisor.

I-Surfaces with One T-Singularity
Marco Franciosi

I will report on joint work with R.Pardini, J.Rana, S. Rollenske, on I-surfaces, i.e., stable surfaces with K^2=1, p_g=2 and q=0. I will give a review of past results on Gorenstein surfaces and I will consider normal I-surfaces with a T-singular point, giving applications to the analysis of the KSBA compactification of Gieseker's moduli space of canonical surfaces with K^2=1 and Euler characteristic =3.

Rationality, Unirationality and Infinite Transitivity
Fedor Bogomolov

In my talk I want to touch several classical approaches to the proof of (stable) rationality and relate the problem of unirationality and infinite transitivity.

Variation of Extension Data of Limit Mixed Hodge Structures
Mark Green

TBA The extension data in limit mixed Hodge structures carries interesting geometric information. I will give some examples of this and discuss restrictions on how this data can vary in a family from joint work with Phillip Griffiths and Colleen Robles. These restrictions go involve a mix of Lie theory and the integral structure.

Deformations of Cusp Singularities of Algebraic Surfaces and Mirror Symmetry
Paul Hacking

Cusp singularities arise on the degenerate surfaces at the boundary of the functorial compactification of the moduli space of surfaces of general type. The Milnor fiber of a smoothing of a cusp singularity is mirror to a log Calabi--Yau surface --- a pair (Y,D) consisting of a projective surface Y and a normal crossing divisor D such that K_Y+D=0. This leads to a precise conjectural description of the smoothing components of the deformation space of a cusp singularity in terms of the Kahler cones of the mirror surfaces and their monodromy groups. Based on joint work with Mark Gross and Sean Keel; Ailsa Keating; and graduate students Jennifer Li and Angelica Simonetti.

Compactifications of Moduli — Geometry vs. Hodge Theory
Radu Laza

The construction and compactification of moduli spaces is a topic of great interest in algebraic geometry. Typically, there are several options for pursuing this problem. In this talk, I will focus on comparing two approaches to the compactification problem: a geometric approach vs. a Hodge theoretic approach. I will give some examples and illustrate some issues in “the classical case” (corresponding to VHS of abelian variety type and of K3 type respectively). I will then discuss a project (joint with M. Green, P. Griffiths, and C. Robles) to analyze the possibly simplest non-classical case, namely the H and I-surfaces (surfaces of general type with p_g=2).

Deformations of Semi-Smooth Varieties

Rita Pardini

A variety X is semi-smooth if locally in the e'tale topology its singularities are either double crossing points (xy=0) or pinch points (x^2-y^2z=0). Alternatively, X is semi-smooth if it can be obtained from a smooth variety X' by gluing it along a smooth divisor D' via an involution g of D'. We describe explicitly in terms of the triple (X',D', g) the two sheaves on X that control its deformation theory, that is, the tangent sheaf T_X and the sheaf T^1_X:=ext^1(\Omega_X,O_X). As an application, we discuss the smoothability of the semi-smooth Godeaux surfaces (K^2=1, p_g=q=0). This is joint work with Barbara Fantechi and Marco Franciosi.

Top Weight Cohomology of M_g

Sam Payne

TBA I will discuss an approach to studying the top-graded piece of the weight filtration on open moduli spaces with suitable toroidal compactifications, inspired by tropical and nonarchimedean analytic geometry. One application of this approach is the recent proof, joint with Chan and Galatius, that the dimension of H^{4g-6}(M_g, Q) grows exponentially with g. This growth was unexpected and disproves conjectures of Church-Farb-Putman and Kontsevich.

Divisors in the Moduli Space of Surfaces

Julie Rana

The KSBA moduli space of stable surfaces is a natural compactification of Gieseker's moduli space of surfaces of general type. In contrast with the moduli space of curves, there are very few examples of divisors in KSBA moduli spaces. I will give examples of divisors corresponding to surfaces with cyclic quotient singularities. I will also discuss bounds that help to narrow the search for singular surfaces. Parts of my talk will touch on joint work with Urz\'ua and, separately, with Franciosi, Pardini, and Rollenske.

Completions of Period Maps
Colleen Robles

I will describe (up to finite data) a completion/extension of a period map. The image of the extended period map is a compact Moishezon variety. (The "up to finite data" statement is due to the fact that we work with the Stein factorization of the period map. This allows us to replace the period map with a finite cover that has *connected* fibres.) This is joint work with M. Green and P. Griffiths.

This result is part of a project (with MG, PG and Radu Laza) to construct completions of period mappings, and to apply those completions to study moduli. In the classical case (including ppav and K3 surfaces) we have "minimal" and "maximal" compactifications of the period space (the work of Satake—Baily—Borel and Ash—Mumford—Rapoport—Tai, respectively). And Borel's extension theorem yields a completion of the period map to the minimal SBB compactification; in contrast the existence of an extension to the maximal AMRT compactification is a subtle problem, and the extension may not exist.

The compactified image here has the flavor of the AMRT construction in the sense that it encodes the maximal amount of Hodge theoretical information (similar to the work of Kato—Usui). So it is striking (in comparison with the classical case), that we also obtain an extension.

The key technical inputs here are new results on the "global structure" of period maps at infinity. Loosely one may think of these results as extending consequences of Schmid's nilpotent orbit theorem from local coordinate neighborhoods at infinity to larger neighborhoods. (These larger neighborhoods contain compact varieties: the period map remains proper when restricted to these sets.) Informally what one obtains are period matrix representations of the period map over these sets. This structure, along with the infinitesimal period relation and the work of Cattani—Deligne—Kaplan, allows us to apply Grauert's result on the holomorphic equivalence relations to obtain the main result.

Stratifications in the Moduli Space of Stable Surfaces
Sönke Rollenske

The Gieseker moduli space of surfaces of general type admits a modular compactification, the moduli space of stable surfaces. Our knowledge about the "new" surfaces in the boundary is still limited and I will discuss different possibilities to organise them, in particular a Hodge-theoretic approach proposed by Green, Griffiths, Laza, and Robles. Everything will be illustrated with examples and many pictures. Based on joint work with B. Anthes, M. Franciosi, R. Pardini.

Fixed Loci of Anti-Symplectic Involutions
Giulia Sacca

It is known that to some Fano manifolds whose cohomology looks like that of a K3 surface, one can associate via geometric constructions examples of hyperkahler manifolds. In this talk I will report on the first steps of a program whose aim is to reverse this construction: starting from a hyperkahler manifold how to recover geometrically a Fano manifold? This is joint work with L. Flapan, E. Macrì, and K. O'Grady.

Logarithmic Resolution of Singularities
Michael Temkin

I will talk about a recent series of works with Abramovich and Wlodarczyk, where a logarithmic analogue of the classical resolution of singularities of schemes in characteristic zero is constructed. Already for usual schemes, the logarithmic algorithm is faster and more functorial, though as a price one has to work with log smooth ambient orbifolds rather than smooth ambient manifolds. But the main achievement is that essentially the same algorithm resolves log schemes and even morphisms of log schemes, yielding a major generalization of various semistable reduction theorems.

On Wormholes in the Moduli Space of Surfaces
Giancarlo Urzúa

The talk I will describe a certain wormholing phenomena that takes place in the Kollár--Shepherd-Barron--Alexeev (KSBA) compactication of the moduli space of surfaces of general type. It occurs because of the appearance of particular extremal P-resolutions in surfaces on the KBSA boundary. I will state a general wormhole conjecture, and I will show we can prove it for a wide range of cases. There will be an emphasis on explaining open questions.