

# Topological & Geometric-Recursion

Conference  
IMSA Miami, 2-8 February 2020

## Program

### Monday 3 February 2020

|                       |                             |
|-----------------------|-----------------------------|
| 9:30 am – 10:30 am    | <b>Bertrand Eynard</b>      |
| 11:00 am – 12:00 noon | <b>Stavros Garoufalidis</b> |
| 2:00 pm – 3:00 pm     | <b>Chiu-Chu Melissa Liu</b> |
| 3:30 pm – 4:30 pm     | <b>Ralph Kaufmann</b>       |
| 5:00 pm – 6:00 pm     | <b>Danilo Lewanski</b>      |

### Tuesday 4 February 2020

|                       |                         |
|-----------------------|-------------------------|
| 9:30 am – 10:30 am    | <b>Maxim Kontsevich</b> |
| 11:00 am – 12:00 noon | <b>Yan Soibelman</b>    |
| 2:00 pm – 3:00 pm     | <b>Ernesto Lupercio</b> |
| 3:30 pm – 4:30 pm     | <b>Du Pei</b>           |
| 5:00 pm – 6:00 pm     | <b>Rinat Kashaev</b>    |

### Wednesday 5 February 2020

|                       |                                  |
|-----------------------|----------------------------------|
| 9:30 am – 10:30 am    | <b>Jørgen Ellegaard Andersen</b> |
| 11:00 am – 12:00 noon | <b>Piotr Sulkowski</b>           |
| 2:00 pm – 3:00 pm     | <b>Alessandro Giacchetto</b>     |
| 3:30 pm – 4:30 pm     | <b>Séverin Charbonnier</b>       |
| 5:00 pm – 6:00 pm     | <b>Paolo Gregori</b>             |

### Thursday 6 February 2020

|                       |                          |
|-----------------------|--------------------------|
| 9:30 am – 10:30 am    | <b>Motohico Mulase</b>   |
| 11:00 am – 12:00 noon | <b>Olivia Dumitrescu</b> |
| 2:00 pm – 3:00 pm     | <b>Sergei Gukov</b>      |
| 3:30 pm – 4:30 pm     | <b>Leonid Chekhov</b>    |
| 5:00pm – 6:00pm       | <b>Gäetan Borot</b>      |

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## ABSTRACTS

### **Geometri Recursion**

*Jørgen Ellegaard Andersen*

We shall review the geometric recursion and its relation to topological recursion. In particular, we shall consider the target theory of continuous functions on Teichmüller spaces and we shall exhibit a number of classes of mapping class group invariant functions, which satisfies the geometric recursion. Many of these classes of functions are integrable over moduli spaces and we prove that these averages over moduli spaces satisfies topological recursion. The talk will end with a discussion of open Geometric Recursion. The talk is based on joint work with Borot and Orantin.

### **Statistics of Length of Curves on Surfaces and Masur-Veech Volumes**

*Gaetan Borot*

I will describe what the geometric recursion (see Andersen's talk) on the hyperbolic and the combinatorial Teichmüller space (see Charbonnier's and Giacchetto's talks) can tell us on the length statistics of multicurves and Masur-Veech volumes on the top stratum of the moduli space of quadratic differentials.

Based on joint works with Jørgen Andersen, Severin Charbonnier, Vincent Delecroix, Alessandro Giacchetto, Danilo Lewanski, Nicolas Orantin and Campbell Wheeler.

### **Geometric Recursion for Combinatorial Teichmüller Spaces and Applications**

*Séverin Charbonnier*

We harvest the fruits of the setup described in Alessandro Giacchetto's talk : first, we define Geometric Recursion with the target theory of measurable functions on combinatorial Teichmüller spaces. As an example for the whole talk, a combinatorial version of Mirzakhani-McShane identity proves that the constant functions 1 on the combinatorial Teichmüller spaces satisfy GR. Second, by assuming stronger admissibility conditions for initial data of GR, the integrals over the combinatorial moduli spaces satisfy Topological Recursion. The application of this result to our example yields another proof of Witten-Kontsevich theorem, parallel to Mirzakhani's proof. Third, the integral structures of the combinatorial Teichmüller space allow to define a discrete version of Topological Recursion, which, in our example, reproduces Norbury's polynomials for integral points of the combinatorial moduli spaces. Last, we shall mention other applications of GR in this setup. This is joint work with Jørgen Ellegaard Andersen, Gaëtan Borot, Alessandro Giacchetto, Danilo Lewański and Campbell Wheeler.

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## Classical and Quantum Algebras of $SSL_n$ Monodromies and the Groupoid Action

*Leonid Chekhov*

We use Fock-Goncharov coordinates on Riemann surfaces  $\Sigma_{g,s,n}$  with holes and  $n > 0$  marked points on hole boundaries to construct classical and quantum Darboux-coordinate representations for higher-dimension symplectic leaves of the groupoid of upper-triangular matrices. I will describe the related braid-group action and generalizations of this construction to arbitrary planar acyclic networks together with possible links to geometric recursion. (Based on joint work with Misha Shapiro.)

## Topological Recursion, Virasoro-W-Algebra Constraints, and Kontsevich-Soibelman

*Bertrand Eynard*

We recall how to write Virasoro and W-algebra constraints associated to a spectral curve in the language of topological recursion. We show how to recover in this language the Kontsevich-Soibelman quadratic operators annihilating the partition function. And we discuss the relationship to CFT.

## Resurgence of Faddeev's Quantum Dilogarithm Function

*Stavros Garoufalidis*

We prove that the Borel resummation of a formal power series solution of a linear difference equation produces Faddeev's quantum dilogarithm. Joint work with Rinat Kashaev from 2006.

## On the Kontsevich Geometry of the Combinatorial Teichmüller Space

*Alessandro Giacchetto*

In the early '80s, a combinatorial description of the moduli spaces of curves was discovered, which led to remarkable results about its topology. Among such, the proof of Witten's conjecture by Kontsevich. Inspired by analogue results in the hyperbolic setting, we describe the combinatorial Teichmüller space parametrising marked metric ribbon graphs on a surface. We introduce global Fenchel-Nielsen coordinates, and prove a formula for the Kontsevich symplectic form, which is an analog of Wolpert's formula for the Weil-Petersson form on the ordinary Teichmüller space. To conclude, we describe a method for integrating geometric functions over the combinatorial moduli spaces. Applications of this set-up will be discussed during Séverin Charbonnier's talk. The talk is based on a joint work in progress with J.E. Andersen, G. Borot, S. Charbonnier, D. Lewański and C. Wheeler.

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## **Asymptotics and Resurgence in Weil-Petersson Volumes**

*Paolo Gregori*

Following a recent revival of interest in 2-dimensional gravity due to the holographic properties of Jackiw-Teitelboim gravity, we investigate the non-perturbative properties of such models using the tools offered by Resurgence. This leads to non-trivial results concerning the asymptotics of Weil-Petersson volumes and the instanton contributions to 2-dimensional topological gravity.

## **Topological Recursion: from A to Z**

*Sergei Gukov*

Abstract: TBA

## **On the Spectral Problem of a Three Term Difference Operator**

*Rinat Kashaev*

We address the spectral problem of the quantum mechanical operator associated to the quantized mirror curve of the toric (almost) del Pezzo Calabi—Yau threefold called local  $P^2$  in the case of complex values of Planck's constant. This is a joint work with Sergey Sergeev.

## **Prequels and Possible Sequels to Topological Recursion**

*Ralph Kaufmann*

This talk will be in three parts. In the first, we recall a prequel to Topological recursion that appeared in our considerations of Higher Weil Peterson volumes and logarithmic field theories in collaboration with Kontsevich Manin and Zagier in the mid 90s. In the second part, we first introduce the structure of Hopf algebras of graphs and more generally Bi-algebras and Hopf algebras from Feynman categories and consider their infinitesimal aka. logarithmic structure. These are results recently published with Galvez and Tonks. In the third part, we combine these two theories using work with Berger which shows how to obtain moduli spaces from discrete data through cubical complexes. We then match this with topological recursion making several conjectures about generalization and future directions.

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## Topological Recursion for Hurwitz Theory and Intersection Theory of Moduli Spaces of Curves

*Danilo Lewański*

Since the foundation of topological recursion (TR) theory, Hurwitz theory provided one of the largest set of explicitly worked out examples of enumerative geometric problems generated by TR. Hurwitz enumerative problems consist in counting branched covers of a base Riemann surface with prescribed ramification profiles: the nature of the ramifications defines the problem. Another motivation to study Hurwitz theory concerns the intersection theory of the moduli spaces of curves. On the one hand, the numbers generated by TR are intrinsically related to the moduli spaces of curves; on the other hand, the celebrated ELSV formula expresses Hurwitz numbers in terms of intersections of the Hodge class with psi classes. Since the original ELSV, many more ELSV-type formulae appeared in the literature, involving more complicated classes in place of the Hodge class, and establishing, as a result, a link between the different natures of the ramifications and cohomological field theories. We will go through this interplay, focusing on a few conjectures and summarizing the state-of-the-art.

## Is there Geometric Recursion for The Moduli Space of Toric Varieties?

*Ernesto Lupercio*

In this talk, I present the moduli space of quantum toric varieties stressing the analogies that it exhibits with the case most often considered in geometric recursion. This is joint work with Katzarkov, Meersseman and Verjovsky.

## Open/Relative Correspondence and Topological Recursion

*Chiu-Chu Melissa Liu*

We will describe an open/relative correspondence between (1) open Gromov-Witten invariants counting holomorphic curves in a symplectic manifold  $X$  bounded by a Lagrangian submanifold  $L$  which is rationally homotopic to a circle, and (2) relative Gromov-Witten invariants of a pair  $(X, D)$  where  $X$  is a symplectic manifold and  $D$  is a symplectic submanifold of codimension two. In many cases these invariants satisfy the topological recursion.

## BPS states and 3d TQFTs

*Du Pei*

The problem of counting BPS states in string theory leads to new invariants of 3-manifolds and new three-dimensional topological quantum field theories (3d TQFTs). In this talk, I will review some recent progress in this direction, and how these new TQFTs are related to each other and to more familiar TQFTs such as Chern-Simons theories.

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## **Analytic Wall-Crossing Structures and Resurgence.**

*Yan Soibelman*

Few years ago in a joint paper with Maxim Kontsevich (see arXiv:1303.3253) we proposed the notion of wall-crossing structure (WCS for short) as a way to axiomatize our generalization of Donaldson-Thomas (=BPS) invariants and wall-crossing formulas for them (see arXiv: 0811.2435, arXiv:1006.2706 for the foundations). A special case of the WCS is the notion of stability data on a graded Lie algebra. It turns out that various versions of the notion of stability data on the Lie algebra of vector fields on a complex torus are responsible for algebraicity and resurgence of generating series which appear in different areas of mathematics and physics. In this talk I am going to discuss the notion of analytic stability data. It provides a formalism for studying resurgence. If time permits I will explain how to use the new language for explaining the so-called Ecalle-Voronin resurgence.

## **A-Polynomials for Quivers and Topological Recursion**

*Piotr Sułkowski*

Abstract: TBA