Simons Collaboration on Homological Mirror Symmetry

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Homological Mirror Symmetry and Topological Recursion, Miami, January 27-February 1, 2020

Organizers: D. Auroux, L. Katzarkov, M. Kontsevich, E. Lupercio, T. Pantev

This event is partially supported by the Simons Collaboration on Homological Mirror Symmetry, with the assistance of the University of Miami Department of Mathematics. The programme will feature mini-courses and a range of research talks in various areas of homological mirror symmetry and related topics.

This conference will be followed by a focus semester on Topological and Geometric Recursion in Interaction with Resurgence (https://www.imsa.miami.edu/events/spring-2020-emphasis-semester/index.html) at the University of Miami's Institute of Mathematical Sciences in the Americas.

This year's mini-courses will be given by Maxim Kontsevich, Bertrand Eynard, Denis Auroux, and Jørgen Ellegaard Andersen.

Venue: The conference is scheduled to take place at the Marriott Courtyard Coral Gables (http://www.marriott.com/hotels/travel/miagb-courtyard-miami-coral-gables/), from Monday morning until Saturday lunchtime.

Airport: Miami International Airport is about 7 miles away. The most convenient way to reach the conference venue is to take a taxi. Participants expecting reimbursement: please keep all your original boarding passes since they may be needed for reimbursement.

Speakers:

Maxim Kontsevich
Jørgen Ellegaard Andersen
Denis Auroux
Bertrand Eynard

Mohammed Abouzaid
Matthew Ballard
Roman Bezrukavnikov
Ron Donagi
Yu-Wei Fan
Benjamin Gammage
Alexander Goncharov
Sergey Gukov
Yuan-Pin Lee
Ernesto Lupercio
Tony Pantev
Alexander Polishchuk
Artan Sheshmani
Bernd Siebert
Yan Soibelman
Abigail Ward

Registration: There will be a registration fee of $40 to cover the cost of refreshments at the conference. A registration web form has been set up in order to allow participants to register, pay the registration fee, and request a hotel reservation. Please register here. (http://www.math.miami.edu/hms2020/)

Tentative schedule

Monday January 27

9:30-10:30	**M. Kontsevich:** *Quantum spectrum in algebraic geometry I* (Video) (https://youtu.be/Dz14DBtnnKo)
11:00-	**Y. Soibelman:** *Cohomological Hall algebras, Nekrasov instantons, and Coulomb branches of 3d N=4*

12:00	*gauge theories* (Slides) (https://schms.math.berkeley.edu/wp-content/uploads/sites/6/2020/01/Y.-Soibelman-27-01-20-Cohomological-Hall-algebras-Nekrasov-instantons-and-Coulomb-branches.pdf) (Video) (https://youtu.be/xsLKtwlFGPc)
2:00-3:00	**D. Auroux:** *Mirrors of curves and their Fukaya categories I* (Video) (https://youtu.be/1FoZzncA_m0)
3:30-4:30	**M. Abouzaid:** *Floer homotopy without spectra* (Video) (https://youtu.be/h9R2clf3KZM)
5:00-6:00	**R. Donagi:** *Families of Hitchin systems and SCFTs of class S* (Video) (https://youtu.be/-Bxn88tbUmg)

Tuesday January 28

9:30-10:30	**D. Auroux:** *Mirrors of curves and their Fukaya categories II*
11:00-12:00	**B. Siebert:** *Toward homological mirror symmetry for intrinsic mirror pairs*
2:00-3:00	**M. Kontsevich:** *Quantum spectrum in algebraic geometry II*
3:30-4:30	**A. Polishchuk:** *Homological mirror symmetry for higher dimensional pairs of pants*
5:00-6:00	**R. Bezrukavnikov:** *A local version of HMS for parabolic Hitchin space*

Wednesday January 29

9:30-10:30	**D. Auroux:** *Mirrors of curves and their Fukaya categories III*
11:00-12:00	**B. Gammage:** *Mirror symmetry near large volume limit*
2:00-3:00	**A. Goncharov:** *Quantization of moduli spaces of local systems at roots of unity and its applications*
3:30-4:30	**A. Ward:** *Homological mirror symmetry for some non-Kähler elliptic surfaces*
5:00-6:00	**A. Sheshmani:** *Higher rank flag sheaves on projective surfaces, and relations to Vafa-Witten and Kapustin-Witten invariants for complex 3 and 4 folds*

Thursday January 30

9:30-10:30	**B. Eynard:** *Topological recursion: a B-model method to compute Gromov-Witten invariants and enumerative geometry I*
11:00-12:00	**Y.-P. Lee:** *Functoriality in Gromov-Witten theory*
2:00-3:00	**M. Kontsevich:** *Quantum spectrum in algebraic geometry III*
3:30-4:30	**M. Ballard:** *Exceptional collections and rationality*
	Reception at IMSA

Friday January 31

9:30-10:30	**T. Pantev:** *The pushforward theorem and its applications*
11:00-12:00	**J.E. Andersen:** *Geometric recursion I*
2:00-3:00	**B. Eynard:** *Topological recursion: a B-model method to compute Gromov-Witten invariants and enumerative geometry II*
3:30-4:30	**S. Gukov:** *A proposal for open DT invariants*
5:00-6:00	**E. Lupercio:** *Remarks on the moduli space in quantum toric geometry*
	Conference dinner

Saturday February 1

9:00-10:00	**B. Eynard:** *Topological recursion: a B-model method to compute Gromov-Witten invariants and enumerative geometry III*
10:30-	**Y.-W. Fan:** *New rational cubic fourfolds via Cremona transformation*

11:30	
12:00-1:00	**J.E. Andersen:** *Geometric recursion II*

Titles and Abstracts

J.E. Andersen: *Geometric recursion*

Abstract: We shall review the geometric recursion and its relation to topological recursion. In particular, we shall consider the target theory of continuous functions on Teichmüller spaces and we shall exhibit a number of classes of mapping class group invariant functions, which satisfies the geometric recursion. Many of these classes of functions are integrable over moduli spaces and we prove that these averages over moduli spaces satisfies topological recursion. The talk will end with some future perspectives of applications of geometric recursion, in particular discussions of other target theories with relevance for Gromov-Witten invariants and Fukaya Categories. The construction of geometric recursion and the results relating it to topological recursion is joint work with Borot and Orantin.

D. Auroux: *Mirrors of curves and their Fukaya categories*

Abstract: These talks will focus on homological mirror symmetry for curves, and in particular, the symplectic geometry of their mirrors (i.e., comparing coherent sheaves on a curve to a suitable Fukaya category of the mirror). In the first talk, we will review the construction of Landau-Ginzburg models mirror to curves in $(\mathbf{C}^*)^2$ or in toric surfaces, and the notion of fiberwise wrapped Fukaya category, as well as computations and verifications of HMS in this framework (this is joint work with Mohammed Abouzaid). We will also discuss the geometric relationship between the smooth fiber of the Landau-Ginzburg model, its singular fiber, and its total space, and the corresponding functors between their Fukaya categories (touching on work in progress by Maxim Jeffs). These results also apply to hypersurfaces in higher-dimensional toric varieties, in abelian varieties (Catherine Cannizzo's thesis), and to complete intersections.

We will then discuss more speculative ways of viewing the symplectic geometry of the mirror from a lower-dimensional perspective: first in terms of "tropical Lagrangians" in the fiber of the Landau-Ginzburg model (using ideas of Andrew Hanlon and Jeff Hicks), and then in terms of geometry inside the critical locus (work in progress with Alexander Efimov and Ludmil Katzarkov). For mirrors of curves, this leads to a new flavor of Lagrangian Floer theory in trivalent configurations of Riemann surfaces, which we will describe in detail as well as the manner in which it relates to the geometry of the curve.

M. Ballard: *Exceptional collections and rationality*

Abstract: The general expectation, attributed to Orlov, is that a smooth projective variety with a full exceptional collection must be rational over its base field. We will discuss this question over non-closed fields. We will show that there exists smooth projective geometrically rational 3-folds which possess full etale-exceptional collections (where End(E) is a finite separable extension of the base field) but not any points over the base field. In the other direction, we will show the expectation holds for smooth projective toric varieties over any base field: if a smooth projective (neutral) toric variety over k possesses a full k-exceptional collection then it is in fact k-rational. This is joint work with A. Duncan, A. Lamarche, and P. McFaddin.

R. Bezrukavnikov: *A local version of HMS for parabolic Hitchin space*

Abstract: I will describe joint project(s) with Michael McBreen and Zhiwei Yun relating coherent sheaves on a Springer fiber to microlocal sheaves on an affine Springer fiber for the dual group. This can be thought of as a local counterpart of the mirror duality between parabolic Hitchin spaces.

R. Donagi: *Families of Hitchin systems and SCFTs of class S*

Abstract: Superconformal theories of class S in 4 dimensions are obtained from a 6-dimensional theory by compactifying on a punctured Riemann surface. The moduli of such theories is the Deligne-Mumford compactification of the moduli of punctured Riemann surfaces, and many aspects of these theories, such as their Coulomb branches and the appearance of weakly coupled gauge groups, are encoded in the global behavior of the family of Hitchin systems over this moduli space. We explore several aspects of this correspondence.

B. Eynard: *Topological recursion: a B-model method to compute Gromov-Witten invariants and enumerative geometry*

Y.-W. Fan: *New rational cubic fourfolds via Cremona transformation*

Abstract: It is conjectured that two cubic fourfolds are birational if their associated K3 categories are equivalent. We prove this conjecture in the case when one of the cubic fourfolds contains a Veronese surface. The main technique is Cremona transformation on P^5. Using the same technique, we find new rational cubic fourfolds. Joint work with Kuan-Wen Lai.

B. Gammage: *Mirror symmetry near large volume limit*

Abstract: The Gross-Siebert program begins with a singular "large-complex-structure-limit" variety equipped with some extra structure, including a smooth log structure outside codimension 2 and an SYZ moment map, and deforms it to a smooth variety. We begin by describing a mirror large-volume-limit variety and construct a "large-volume SYZ" map mirror to the LCS moment map. Combining this structure with Viterbo functoriality, B-side étale descent for deformations, and calculations in local models, we describe an approach to proving a homological mirror symmetry equivalence relating a compactification of the LVL variety to a smoothing of the Gross-Siebert central fiber. This is joint work with Vivek Shende.

A. Goncharov: *Quantization of moduli spaces of local systems at roots of unity and its applications*

Abstract: We describe the quantization of moduli spaces of G-local systems on decorated surfaces at roots of unity, including the modular functor conjecture. It provides a categorical structure generalising modular tensor categories. Applications to quantum topology and representations of DeConcini-Kac quantum groups at roots of unity will be discussed.

S. Gukov: *A proposal for open DT invariants*

M. Kontsevich: *Quantum spectrum in algebraic geometry*

Abstract: The name 'Quantum Spectrum' in the title refers (for a compact symplectic manifold) to the spectrum of the operator of quantum multiplication by the first Chern class of the tangent bundle, considered as a multi-valued function on the Frobenius manifold encoding genus=0 Gromov-Witten invariants.

In the algebraic case, for complex projective varieties, one of the greatest not yet understood mysteries known from early days of mirror symmetry, is a conjectural relation (by S.Barannikov, B.Dubrovin and myself) between the quantum spectrum and semi-orthogonal decompositions of the derived category of coherent sheaves. We are very far from even approaching this problem in complete generality, beyond particular explicit examples.

The main goal of the lectures is the formulation of quantum blow-up formula which seems to be totally within reach, leading to numerous consequences in the mainstream algebraic geometry.

One application is the construction of a new very strong birational invariant (also for non-algebraically closed base fields). In particular, one can deduce non-rationality of a generic cubic 4-fold over complex numbers. Another application is a construction of a new exotic motivic measure. More speculatively, one can have a relation to Minimal Program, a categorification of intersection cohomology for varieties with canonical singularities, and an obstruction to the strong resolution of singularities in positive characteristic.

Y.-P. Lee: *Functoriality in Gromov-Witten theory*

Abstract: I will discuss the functoriality problem in Gromov–Witten theory, and explain a series of results in this context, including quantum Lefschetz, quantum Leray-Hirsch, as well as the functoriality with respect to birational transformations.

E. Lupercio: *Remarks on the moduli space in quantum toric geometry*

Abstract: In this talk I will define the main objects appearing in quantum toric geometry (which quantize classical toric geometry): Quantum toric stacks and their moduli spaces. Then, I will explain how to put a natural "complex structure of to homotopy" on the moduli space.

T. Pantev: *The pushforward theorem and its applications*

Abstract: I will discuss the notion of a relative shifted symplectic structure along the stalks of a constructible sheaf of derived stacks on a stratified space. I will describe a general pushforward theorem producing relative shifted symplectic forms and will explain explicit techniques for computing such forms. As an application I will describe a universal construction of Poisson structures on derived moduli of Stokes data on smooth varieties and will explain how symplectic leaves arise from fixing irregular types and local formal monodromies at infinity. This is a joint work with Dima Arinkin and Bertrand Toën.

A. Polishchuk: *Homological mirror symmetry for higher dimensional pairs of pants*

Abstract: I will describe joint work with Yanki Lekili. We prove a homological mirror symmetry equivalence between the wrapped Fukaya category of the higher dimensional pair-of-pants and the derived category of coherent sheaves on the union of coordinate hyperplanes in the affine space. Our method is to add a stop and identify the corresponding partially wrapped Fukaya category with a categorical resolution on the B-side.

A. Sheshmani: *Higher rank flag sheaves on projective surfaces, and relations to Vafa-Witten and Kapustin-Witten invariants for complex 3 and 4 folds*

Abstract: We study moduli space of holomorphic triples , composed of (possibly rank) torsion-free sheaves and a holomorphic map between them, over a smooth complex projective surface . The triples are equipped with a Schmitt stability condition. We prove that when the Schmitt stability parameter becomes sufficiently large, the moduli space of triples benefits from having a perfect relative and absolute obstruction theory in some cases (depending on Chern character of). We further generalize our construction to higher-length flags of higher rank sheaves by gluing triple moduli spaces, and extend earlier work, with Gholampur and Yau, where the obstruction theory of nested Hilbert schemes over the surface was studied. Here we extend the earlier results to the moduli space of flags , where the maps are injective (by stability). There is a connection, by wall-crossing in the master space, developed by Mochizuki, between the theory of such higher rank flags, and the theory of Higgs pairs on the surface, which provides the means to relate the flag invariants to the local DT invariants of a threefold given by a line bundle over the surface, . The latter DT invariants, when L is the canonical bundle of S, contribute to Vafa-Witten invariants. Joint work with Shing-Tung Yau, arXiv:1911.00124. If the time permits, I will also talk about some more generalizations of this construction to local surface 4-folds and connections to Kapustin-Witten theory.

B. Siebert: *Toward homological mirror symmetry for intrinsic mirror pairs*

Abstract: I will report on ongoing work with Tim Perutz aiming at proving a version of homological mirror symmetry for the intrinsic mirror pairs constructed with Mark Gross via a logarithmic version of the quantum cohomology ring. The main point is a comparison of this algebraic-geometrically defined ring with a version of the degree zero symplectic cohomology using John Pardon's VFC machinery.

Y. Soibelman: *Cohomological Hall algebras, Nekrasov instantons, and Coulomb branches of 3d N=4 gauge theories*

Abstract: Talk consists of two parts. In the first one I am going to discuss the role of Cohomological Hall algebras in the generalized AGT conjecture. It gives a 3-dimensional Calabi-Yau point of view on some results in geometric representation theory related to moduli spaces of instantons. The second part of the talk is devoted to some speculations related to 3-dimensional gauge theories. The speculations are devoted to Cohomological Hall algebras as well as generalized Riemann-Hilbert correspondence.

A. Ward: *Homological mirror symmetry for some non-Kähler elliptic surfaces*

Abstract: We will first present homological mirror symmetry results for elliptic surfaces which are constructed by performing two logarithmic transformations to the product of $\mathbf{P}^1$ with an elliptic curve, a class of surfaces which includes the classical Hopf surface ($S^1 \times S^3$). We will then use this work, along with work of Auroux, Efimov and Katzarkov on the Fukaya category of singular curves, to give some speculations on a mirror operation to the logarithmic transformation and its potential applications.