Recent Applications of the Theory of O-Minimal Structures to Various Questions in Hodge Theory

Virtual Conference

IMSA Miami, 16-20 November 2020

Program

Monday 16 November 2020

9:00 am – 10:00 am	**Dr. Boris Zilber**
10:15 am – 11:15 am	**Dr. Bruno Kingler**
2:00 pm – 3:00 pm	**Dr. Mark Andrea de Cataldo**
3:30 pm – 4:30 pm	**Dr. Mihnea Popa**

Tuesday 17 November 2020

9:00 am – 10:00 am	**Dr. Boris Zilber**
10:15 am – 11:15 am	**Dr. Bruno Kingler**
2:00 pm – 3:00 pm	**Dr. Jacob Tsimerman**
3:30 pm – 4:30 pm	**Dr. Christian Schnell**

Wednesday 18 November 2020

9:00 am – 10:00 am	**Dr. Jacob Tsimerman**
10:15 am – 11:15 am	**Dr. Matt Kerr**
2:00 pm – 3:00 pm	**Dr. Patrick Brosnan**
3:30 pm – 4:30 pm	**Dr. Greg Pearlstein**

Thursday 19 November 2020

9:00 am – 10:00 am	**Dr. Radu Laza**
10:15 am – 11:15 am	**Dr. Wilfried Schmid**
2:00 pm – 3:00 pm	**Dr. Tokio Sasaki**
3:30 pm – 4:30 pm	**Dr. Artan Sheshmani**

Friday 20 November 2020

9:00 am – 10:00 am	**Dr. Andrew Harder**
10:15 am – 11:15 am	**Dr. Victor Przyjalkowski**
2:00 pm – 3:00 pm	**Dr. Abhishek Oswal**
3:30 pm – 4:30 pm	**Dr. Paul Horja**

ABSTRACTS

Fixed Points in Toroidal Compactifications and Essential Dimension of Covers

Patrick Brosnan

Essential dimension is a numerical measure of the complexity of algebraic objects invented by J. Buhler and Z. Reichstein in the 90s. Roughly speaking, the essential dimension of an algebraic object is the number of parameters it takes to define the object over a field. For example, by Kummer theory it takes one parameter to define a mu_n torsor, so the essential dimension of the functor of mu_n torsors (or the essential dimension of the group mu_n for short) is 1.

In a preprint from 2019, Farb, Kisin and Wolfson (FKW) prove theorems about the essential dimension of congruence covers of Shimura varieties using arithmetic methods. In many cases, they are able to prove that the congruence covers are incompressible, that is, they are not obtainable by base change from varieties of strictly smaller dimension. In my talk, I will discuss recent work with Najmuddin Fakhruddin, where we recover many (but definitely not all) of the results of FKW, by geometric arguments using a new fixed point theorem. This also allows us to extend the incompressibility results of FKW to Shimura varieties of exceptional type to which the arithmetic methods of FKW do not apply. I will also discuss a general conjecture we make on the essential dimension of congruence covers arising from Hodge theory. (With some caveats, we conjecture roughly that it is equal to the dimension of the image of the period map.)

The P=W conjecture in genus two and the Hodge numbers of O'Grady 10

Mark Andrea de Cataldo

I will report on two projects, with Rapagnetta and Sacca' and with Maulik and Shen, that deal with moduli of sheaves of families of curves and resulting perverse filtrations on their singular cohomology. In the first project, we determine the Betti and Hodge numbers of the irreducible holomorphic symplectic variety known as O'Grady 10. In the second project, we establish the P=W conjecture for the Non Abelian Hodge Theory in arbitrary rank of a curve of genus two, and provide strong evidence in arbitrary rank and genus.

Log Symplectic Pairs and Mixed Hodge Structures

Andrew Harder

A log symplectic pair is a pair (X,Y) consisting of a smooth projective variety X and a divisor Y in X so that there is a non-degenerate log 2-form on X with poles along Y. I will discuss the relationship between log symplectic pairs and degenerations of hyperkaehler varieties, and how this naturally leads to a class of log symplectic pairs called log symplectic pairs of "pure weight". I will discuss results which show that the classification of log symplectic pairs of pure weight is analogous to the classification of log Calabi--Yau surfaces.

D-Modules and Toric Schobers

Paul Horja

Some of the classical mirror symmetry results can be recast using the recent language of perverse sheaves of categories and schobers. In this context, I will explain a proposal for the B-side category in toric homological mirror symmetry along the strata of the characteristic cycle of the associated D-module.

Differential Equations and Mixed Hodge Structures

Matt Kerr

We report on a new development in asymptotic Hodge theory, arising from work of Golyshev-Zagier and Bloch-Vlasenko, and connected to the Gamma Conjectures in Fano/LG-model mirror symmetry. The talk will focus exclusively on the Hodge/period-theoretic aspects through two main examples.Given a variation of Hodge structure M on a Zariski open in P^1, the periods of the limiting mixed Hodge structures at the punctures are interesting invariants of M. More generally, one can try to compute these asymptotic invariants for iterated extensions of M by "Tate objects", which may arise for example from normal functions associated to algebraic cycles.The main point of the talk will be that (with suitable assumptions on M) these invariants are encoded in an entire function called the motivic Gamma function, which is determined by the Picard-Fuchs operator L underlying M. In particular, when L is hypergeometric, this is easy to compute and we get a closed-form answer (and a limiting motive). In the non-hypergeometric setting, it yields predictions for special values of normal functions; this part of the story is joint with V. Golyshev and T. Sasaki.

Tame Geometry and Hodge Theory I & II
Bruno Kingler

In this minicourse I will survey a number of recent applications of tame geometry to several problems related to Hodge theory and periods. After recalling basics on o-minimal structures and their tameness properties, I will discuss the use of tame geometry in proving algebraization results (Pila-Wilkie theorem; o-minimal Chow and GAGA theorems in definable complex analytic geometry); the tameness of period maps and its application to algebraicity of images of period maps; functional transcendence results of Ax-Schanuel type for variations of Hodge structures; and some atypical intersection conjectures in Hodge theory.

Remarks on Degenerations of K-Trivial Varieties
Radu Laza

Due to Kulikov's theorem and its applications, one has a good understanding of the degenerations of K3 surfaces and consequently some understanding of compactifications for moduli of K3 surfaces. In this talk, I will discuss some aspects of higher dimensional analogues of these results. Most of the results will concern Hyperkaehler manifolds, where the picture is quite similar to that for K3 surfaces. I will close with some ideas on how to deal with the more subtle Calabi-Yau case.

A Non-Archimedean Definable Chow Theorem
Abhishek Oswal

Algebraization theorems originating from o-minimality have found some surprising applications in Diophantine geometry and Hodge theory. One such key result is the 'definable Chow theorem' of Peterzil and Starchenko which states that every closed analytic subset of a complex algebraic variety that is also definable an o-minimal structure, is in fact an algebraic subset. This talk will be about a non-archimedean analogue of this result.

Archimedean Height Pairings for Higher Cycles
Greg Pearlstein

By the work of Richard Hain, the archimedean height pairing on ordinary algebraic cycles can be interpreted as an invariant of an associated mixed Hodge structure. In this talk, we will present a similar construction for higher cycles in the Bloch complex. Families of higher cycles produce admissible variations of mixed Hodge structure. We will describe the asymptotic behavior of the height pairing in the case where the associated variation of mixed Hodge structure is Hodge-Tate. This is joint work with J. Burgos Gil and S. Goswami.

Minimal Exponents of Singularities

Mihnea Popa

The minimal exponent of a function is the negative of the largest root of its reduced Bernstein-Sato polynomial. It refines the notion of log canonical threshold, and it is related (sometimes conjecturally) to other interesting objects, for instance the Igusa zeta function. I will describe some results towards understanding minimal exponents, based on viewing them in the context of D-modules and Hodge theory on one hand, and birational geometry on the other.
This is joint work with Mircea Mustata.

Hodge-Theoretic Aspects of Mirror Symmetry

Victor Przyjalkowski

We discuss mirror symmetry for Fano varieties from Hodge theory point of view. We recall the Hodge diamond rotation phenomena for Calabi--Yau varieties. Then we pass to Fano varieties, formulate Katzarkov--Kontsevich--Pantev conjectures and discuss their proofs in low dimensions. Finally, we discuss the mirror P=W conjecture stating mirror correspondence in terms of mixed Hodge structures and discuss its relations with other Hodge mirror symmetry conjectures.

Higher Chow Cycles Arising from Some Laurent Polynomials

Tokio Sasaki

An example of the constructions of Calabi-Yau hypersurface sections in a toric Fano variety is to consider a pencil defined by a Laurent polynomial. We often can construct non-trivial families of higher Chow cycles from rational irreducible components on its base locus. In this talk, we introduce two examples of such a family of higher cycles and significant properties of the associated higher normal functions.

The first one exhibits a B-model side explanation of Golyshev's Apéry constant on some rank one Fano threefolds defined via the quantum recursions. It is an example of the arithmetic mirror conjecture. The second one is defined on general cubic fourfolds containing a plane. Via the identification of the 2-torsion part of the Brauer group of the associated K3 surface and that of the indecomposable cycles, we expect that this family of higher cycles relates to the rationality problem.

Degenerating Complex Variations of Hodge Structure

Christian Schnell

I will explain how one can adapt Schmid's analysis of polarized variations of Hodge structure over the punctured disk to arbitrary polarized COMPLEX variations (with not necessarily quasi-unipotent monodromy). It turns out that one can get to the main results -- Hodge norm estimates, existence of a limiting mixed Hodge structure, convergence of the rescaled period mapping -- more quickly by a different route, and I will try to sketch how this goes.

Unitary Representations of Reductive Lie Groups

Wilfried Schmid

I shall describe a conjecture, and progress towards that conjecture, about unitary representations of reductive Lie groups, using Hodge theory. This is joint work with Kari Vilonen.

Atiyah Class and Sheaf Counting on Local Calabi-Yau 4 Folds

Artan Sheshmani

We discuss Donaldson-Thomas (DT) invariants of torsion sheaves with 2 dimensional support on a smooth projective surface in an ambient non-compact Calabi Yau fourfold given by the total space of a rank 2 bundle on the surface. We prove that in certain cases, when the rank 2 bundle is chosen appropriately, the universal truncated Atiyah class of these codimension 2 sheaves reduces to one, defined over the moduli space of such sheaves realized as torsion codimension 1 sheaves in a noncompact divisor (threefold) embedded in the ambient fourfold. Such reduction property of universal Atiyah class enables us to relate our fourfold DT theory to a reduced DT theory of a threefold and subsequently then to the moduli spaces of sheaves on the base surface. We finally make predictions about modularity of such fourfold invariants when the base surface is an elliptic K3.

O-Minimality and Hodge Theory: Definable GAGA + Griffiths Conjecture I & II

Jacob Tsimerman

In this pair of lectures, we will explain how to develop an o-minimal geometry allowing for nilpotents, that we call "definable analytic spaces". We explain how to use this theory to prove a definable GAGA statement, and how one can use this to prove a conjecture of Griffiths that the images of period maps are algebraic. We will also discuss the analogous o-minimal in the setting of variations of mixed Hodge structures, and a generalization of Griffiths conjecture to this setting.

Model Theory: From Logic to Geometric Stability Theory and O-Minimality I & II

Boris Zilber

I am going to give a general introduction into model theory and its principles with emphasis on o-minimality as a realisation of these principles, and to some extent stability and categoricity theory with a view on algebraic and analytic complex geometry, including some arithmetic aspects.