Irrational Fans in Physics and Mathematics

Virtual Conference
IMSA Miami, 19-21 October 2020

Program

Monday 19 October 2020

8:00 am – 9:00 am	**Dr. Elisa Prato**
9:00 am – 10:00 am	**Dr. Fiametta Battaglia**
10:00 am – 11:00 am	**Dr. Alberto Verjovsky**
11:00 am – 12:00 pm	**Dr. Frank Sottile**

Tuesday 20 October 2020

9:00 am – 10:00 am	**Dr. Michael Polyak**
10:00 am – 11:00 am	**Dr. Ernesto Lupercio**
11:00 am – 12:00 pm	**Mr. Antonie Boivin**

Wednesday 21 October 2020

8:00 am – 9:00 am	**Dr. Hiroaki Ishida**
9:00 am – 10:00 am	**Mr. Roman Krutowski**
10:00 am – 11:00 am	**Dr. Fiametta Battagila**
11:00 am – 12:00 pm	**Dr. Ernesto Lupercio**

ABSTRACTS

Nonrational Toric Geometry II: Quasifolds, Foliations, Combinatorics and One-parameter Families (Minicourse)
Fiametta Battaglia

Toric quasifolds are a class of singular spaces that were introduced first in the symplectic setting (1999) and then, jointly with Battaglia, in the complex setting (2001). They generalize toric varieties to simple convex polytopes that are not rational.

We give a brief historical survey of the process that initially led to toric quasifolds in the symplectic setting, with an emphasis on the core notions of quasilattice, quasirationality and quasitorus. We discuss a number of examples: the quasisphere, Penrose tilings and quasicrystals, regular convex polyhedra, and irrational Hirzebruch surfaces. We also touch on some recent results, obtained jointly with Battaglia, Zaffran and Iglesias-Zemmour.

Minicourse by Dr. Elisa Prato and Dr. Fiametta Battaglia.

Nonrational Toric Geometry III: Quasifolds, Foliations, Combinatorics and One-parameter Families (Minicourse)
Fiametta Battaglia

We illustrate a new viewpoint, introduced jointly with Dan Zaffran: we establish a correspondence between not necessarily rational simplicial fans and certain foliated complex manifolds called LVMB manifolds. Toric quasifolds are then viewed as leaf spaces. In the rational setting, Meersseman and Verjovsky had shown that the leaf space is the classical toric variety. We then give evidence that the rich interplay between toric geometry and combinatorics carries over to our nonrational context, in fact we are able to reformulate Stanley's argument for the proof of the g-Theorem on the combinatorics of simple polytopes.

We continue by describing, from the viewpoint of foliations, a one-parameter family of toric quasifolds contaning all of the Hirzebruch surfaces. This last part is joint work with Elisa Prato and Dan Zaffran.

Minicourse by Dr. Elisa Prato and Dr. Fiametta Battaglia.

Non-Simplicial Quantum Toric Varieties
Antoine Boivin

In this talk, I will describe a construction of quantum toric varieties associated to an arbitrary fan on a finitely generated subgroup of $\mathbb{R}^d$.

Maximal Torus Actions, Equivariant Principal Bundles and Transverse Equivalence
Hiroaki Ishida

In this talk I will explain the one-to-one correspondence between complex manifolds with maximal torus actions (up to an equivalence relation) and irrational fans. I will also explain about the relation between this correspondence and the canonical foliations.

Basic Cohomology of Moment-Angle Manifolds
Roman Krutowski

For any complete fan Σ we may construct a corresponding complex moment-angle $\mathcal{Z}_{\Sigma}$ manifold with canonical holomorphic foliation $\mathcal{F}$ on it. These foliations generalize toric varieties in the case of non-rational fans. From this perspective, basic cohomology rings of these foliations may be regarded as generalized cohomology rings of toric varieties.

During the talk I am going to present results on calculation of basic de Rham and Dolbeault cohomology of these foliations with the main result being the following Davis-Januszkiewicz type formula

$$
H^{*,*}_{\mathcal{F}}(\mathcal{Z}_{\Sigma}) \cong \mathbb{R}[v_1,\ldots,v_m]
/(I_{\Sigma}+J),\quad
v_i \in H^{1,1}_{\mathcal{F}}(\mathcal{Z}_{\Sigma}),
$$

where ideals I_Σ and J are reconstructed from the fan data. These results were conjectured by Fiammetta Battaglia and Dan Zaffran and are proved in joint works with Hiroaki Ishida and Taras Panov.

Quantum Toric Geometry Minicourse I & II
Ernesto Lupercio

In this minicourse I will explain the foundations and motivations behind quantum toric geometry (QTG) QTG is a generalization of classical toric geometry where all tori are replaced by quantum tori thus allowing fans to become irrational. This is joint work with L. Katzarkov, L. Meersseman and A. Verjovsky. Verjovsky's talk is a prerequisite.

Enumerative Geometry and the Quantum Torus
Michael Polyak

Tropical geometry provides a piece-wise linear approach to algebraic geometry. The role of algebraic curves is played by tropical curves - planar metric graphs with certain requirements of balancing, rationality of slopes and integrality. A number of classical enumerative problems can be easily solved by tropical methods.

We consider a generalization of tropical curves, removing requirements of rationality of slopes and integrality and discuss the resulting theory. One of the classical enumerative problems in algebraic geometry is that of counting of complex or real rational curves through a collection of points in a toric variety. We explain this counting procedure as a construction of certain cycles on moduli of rigid pseudotropical curves. Cycles on these moduli turn out to be closely related to Lie algebras. In particular, counting of both complex and real curves is related to the quantum torus Lie algebra. More complicated counting invariants (the so-called Gromov-Witten descendants) are similarly related to the super-Lie structure on the quantum torus.

Nonrational Toric Geometry I: Symplectic Toric Quasifolds (Minicourse)
Elisa Prato

Toric quasifolds are a class of singular spaces that were introduced first in the symplectic setting (1999) and then, jointly with Battaglia, in the complex setting (2001). They generalize toric varieties to simple convex polytopes that are not rational.

We give a brief historical survey of the process that initially led to toric quasifolds in the symplectic setting, with an emphasis on the core notions of quasilattice, quasirationality and quasitorus. We discuss a number of examples: the quasisphere, Penrose tilings and quasicrystals, regular convex polyhedra, and irrational Hirzebruch surfaces. We also touch on some recent results, obtained jointly with Battaglia, Zaffran and Iglesias-Zemmour.

Minicourse by Dr. Elisa Prato and Dr. Fiametta Battaglia.

Irrational Toric Varieties and the Secondary Polytope

Frank Sottile

Classical toric varieties come in two flavours: Normal toric varieties are given by rational fans in R^n. A (not necessarily normal) affine toric variety is given by finite subset A of Z^n. When A is homogeneous, it is projective. Applications of mathematics have long studied the positive real part of a toric variety as the main object, where the points A may be arbitrary points in R^n. For example, in 1963 Birch showed that such an irrational toric variety is homeomorphic to the convex hull of the set A.

Recent work showing that all Hausdorff limits of translates of irrational toric varieties are toric degenerations suggested the need for a theory of irrational toric varieties associated to arbitrary fans in R^n. These are R^n_>-equivariant cell complexes dual to the fan. Among the pleasing parallels with the classical theory is that the space of Hausdorff limits of the irrational projective toric variety of a finite set A in R^n is homeomorphic to the secondary polytope of A.

This talk will sketch this story of irrational toric varieties. It represents work with Garcia-Puente, Zhu, Postinghel, Villamizar, and Pir.

Intersection of Quadrics, Moment-Angle Manifolds, Complex and Quaternionic Manifolds and Convex Polytopes

Alberto Verjovsky

We present an overview of the theory of LVM manifolds and its relation with toric manifolds and present some first steps and comments toward this theory in the quaternionic setting.