Hodge Theory and Rationality

Virtual Conference
IMSA Miami, 5-9 October 2020

<u>Program</u>

Monday 5 October 2020

9:00 am – 10:00 am	**Claire Voisin**
10:15 am – 11:15 am	**Yuri Tschinkel**
2:00 pm – 3:00 pm	**Philip Griffiths**
3:30 pm – 4:30 pm	**Brendan Hassett**
5:00 pm – 6:00pm	**David Favero**

Tuesday 6 October 2020

9:00 am – 10:00 am	**Claire Voisin**
10:15 am – 11:15 am	**Brendan Hassett**
2:00 pm – 3:00 pm	**Alena Pirutka**
3:30 pm – 4:30 pm	**Fedor Bogomolov**

Wednesday 7 October 2020

9:00 am – 10:00 am	**Ivan Cheltsov**
10:15 am – 11:15 am	**Christian Böhning**
2:00 pm – 3:00 pm	**Evgeny Shinder**
3:30 pm – 4:30 pm	**Kyoung-Seog Lee**
5:00 pm – 6:00 pm	**Ryota Mikami**

Thursday 8 October 2020

9:00 am – 10:00 am	**Dimitri Orlov**
10:15 am – 11:15 am	**Alexander Efimov**
2:00 pm – 3:00 pm	**Christian Böhning**
3:30 pm – 4:30 pm	**Matthew Ballard**
4:45 pm – 5:45 pm	**Gregory Pearlstein**

Friday 9 October 2020

9:00 am – 10:00 am	**Alexander Kuznetsov**
10:15 am – 11:30 am	**Alexander Efimov**
2:00 pm – 3:00 pm	**Yu Wei Fan**
3:30 pm – 4:30 pm	**Ludmil Katzarkov**

Abstracts

Rationality and Exceptional Collections for Toric Varieties

Matthew Ballard

A conjecture of Orlov asks whether all varieties with full exceptional collections are rational. We will discuss two interpretations of this question over non-closed fields. The strict one, where End(E)=k is the definition of exceptional for X over k, holds for toric varieties over a general field. The looser one, End(E)/k is a separable extension of fields, does not guarantee the existence of a k-point much less rationality. The work discussed is joint with Alexander Duncan, Alicia Lamarche, and Patrick McFaddin.

A (Biased) Survey of Some Results Inspired by the (Stable) Rationality Problem for Cubic Hypersurfaces – Lecture I

Christian Böhning

We will give an overview of joint work with several co-authors: Asher Auel, Hans-Christian v. Bothmer, Michel van Garrel, Alena Pirutka, Pawel Sosna. Particular emphasis will be placed on recent further developments of the degeneration method involving the p-torsion of the Brauer group in characteristic p or prelog Chow groups (and time permitting both).

Rigid, Not Infinitesimally Rigid Surfaces of General Type with Ample Canonical Bundle – Lecture II

Christian Böhning

In the talk I will report on work in progress, joint with Roberto Pignatelli and Hans-Christian von Bothmer, that concerns the construction of surfaces of general type with ample canonical bundle and Kuranishi space (and possibly also Gieseker moduli space) a non-reduced point. The main tools are configurations of lines and their incidence schemes as well as the theory of abelian covers due to Pardini and others.

I apologize for the thematic remoteness from the main topic of the conference; however, nearly rational varieties help in the construction, and the topic is fun!

Rationality, Unirationality and Infinite Transitivity

Fedor Bogomolov

In my talk I want to touch several classical approaches to the proof of (stable) rationality and relate the problem of unirationality and infinite transitivity.

On One Question of Janos Kollar

Ivan Cheltsov

In 2008, Kollar studied how birational rigidity behave when we extend the field of definition. He asked the following question: if X is a Fano variety over a field F such that X is birationally rigid over the algebraic closure of F, is X birationally rigid over F? Koll\'ar mentioned that he thinks that this is very unlikely, but he does not have a counter-example. His question can be restated for G-Fano varieties as follows: if X is a G-Fano variety for some algebraic group G in Aut(X) such that X is H-birationally rigid for some subgroup H in G, is X G-birationally rigid? I will explain why this question can have a negative answer and will explain how one can try to find a counterexample to it using toric geometry.

Wall Finiteness Obstruction for DG Categories I & II

Alexander Efimov

The classical Wall finiteness obstruction theorem gives a criterion when a finitely dominated CW complex is homotopy equivalent to a finite CW complex. I will explain that an analogue of this result holds for DG categories over a field.

Namely, a homotopically finitely presented DG category is Morita equivalent to a finite cell DG category iff the class of its diagonal bimodule in K_0 is generated by the classes of (box) tensor products of left and right perfect modules. Moreover, finite cell DG categories are (up to Morita equivalence) exactly the quotients of proper DG categories with full exceptional collections, by a subcategory generated by a single object.

As an application, we show that any smooth and proper phantom DG category can be embedded fully faithfully into a proper DG category with a full exceptional collection. The same holds for the derived category of the Barlow surface. This disproves two conjectures of Orlov (since the Barlow surface is not rational). We also obtain that for any smooth proper algebraic variety with a stratification by affine spaces, its derived category admits a fully faithful embedding as above.

We will also sketch the proof of a similar result (Wall finiteness obstruction) for algebras over (colored) DG operads. In this case the class of the diagonal should be replaced by the class of the cotangent complex.

A Look at Fourier-Mukai Transforms and GIT in the Smooth and Singular Settings

David Favero

I will survey recent progress with various collaborators on constructing derived equivalences/semi-orthogonal decompositions from birational geometry. More specifically, I will look at how partial compactifications of group actions lead to Fourier-Mukai functors which compare geometric invariant theory quotients. Passing to the perspective of derived algebraic geometry, I will suggest that deriving these Fourier-Mukai functors could give a general comparison, even in the singular setting.

Completions of Period Mappings

Philip Griffiths

We will define and discuss some properties of and results about the objects in the title.

Rationality for Threefolds Over Non-Closed Fields

Brendan Hassett

We survey recent results on rationality questions for geometrically rational threefolds over non-closed fields due to Benoist-Wittenberg, Kuznetsov-Prokhorov, Tschinkel, etc. as well new approaches via symbol invariants.

Loci of Rational Varieties

Brendan Hassett

Given a family of smooth complex projective varieties, which members are rational? Stably rational? We discuss general structure theorems as well as guiding examples like cubic fourfolds.

NCHS and Rationality

Ludmil Katzarkov

In this talk we will discuss new approached to nonrationality based on Mirror Symmetry.

Intermediate Jacobians of Gushel--Mukai varieties

Alexander Kuznetsov

A Gushel--Mukai variety X is a smooth dimensionally transverse linear section of a 6-dimensional intersection of the cone CGr(2,5) over Gr(2,5) with a quadric. The nontrivial part of the derived category of X is of K3 type when \dim X is even and of a curve-Enriques type when \dim X is odd.

In the talk I will present a description of intermediate Jacobians of GM threefolds and fivefolds as Albanese varieties of the associated double EPW surfaces and discuss birational and categorical counterparts of this phenomenon. This talk is based on joint results with Olivier Debarre and Alex Perry.

Complex Surfaces with $p_g=q=0$ via Birational Geometry, Derived Category and Gauge theory

Kyoung-Seog Lee

Surfaces with $p_g=q=0$ play important roles in the theory of compact complex surfaces. Investigation of these surfaces has a long history and it is still a very active area of research. In the first part of this talk, I will review some parts of the history and recent developments of the theory. Then I will discuss several aspects of these surfaces via birational geometry, derived category and gauge theory.

A Tropical Analog of the Hodge Conjecture for Smooth Algebraic Varieties Over Trivially Valued Fields

Ryota Mikami

Tropical geometry is a combinatorial shadow of algebraic geometry. We propose a tropical approach to problems on cycle class maps such as the Hodge conjecture. In this talk, I would like to explain a proof of a tropical analog of the Hodge conjecture for smooth algebraic varieties over trivially valued fields. The main ingredients are a theorem for general "cohomology theories", developed by many mathematicians, e.g., Quillen, a tropical analog of Milnor K-theory (introduced in this talk), and explicit calculations of tropical cohomology of the trivial line bundles by non-archimedean geometry.

Finite-Dimensional DG Algebras and their Properties

Dimitri Orlov

The talk will focus on finite-dimensional DG algebras and categories of perfect complexes over such DG algebras. These categories can be considered as proper derived noncommutative schemes.

We are going to discuss basic properties of these noncommutative schemes and to establish a connection between such categories and DG categories with (semi)exceptional collections.

Holomorphic Bisectional Curvature and Applications to Deformations and Rigidity of Mixed Hodge Structure

Gregory Pearlstein

The curvature properties of period domains play a fundamental role in Hodge theory. In this talk, I will discuss recent work with Chris Peters on Hodge metrics arising in mixed Hodge theory, and applications to rigidity results for geometric classes of examples.

Birational Geometry and Milnor K-theory
Alena Pirutka

In this talk, we will discuss results on reconstruction of function fields from Milnor K-theory. This is a joint work with A.Cadoret.

Factorization Centers in Dimension Two and the Grothendieck Ring of Varieties
Evgeny Shinder

I explain the concept of factorization centers for birational isomorphisms. The main result is that for smooth projective surfaces over perfect fields these factorization centers for a birational isomorphism f: X->Y are independent of the choice of f and only depend on X and Y. The proof relies on the two dimensional MMP providing the minimal models for surfaces and the link decomposition for morphisms. I explain the relationship to the rationality problem for surfaces and to the structure of the Grothendieck ring of varieties. This is joint work with H.-Y. Lin and S. Zimmermann.

Equivariant Birational Types
Yuri Tschinkel

I will explain new obstructions in equivariant birational geometry, introduced and studied in joint work with Kontsevich, Pestun, Kresch, and Hassett.

Decomposition of the Diagonal and Specialization – Lecture I & II
Claire Voisin

The decomposition of the diagonal is a powerful tool to study the stable irrationality of smooth projective varieties. I will discuss the specialization method, which rests on a certain specialization property of the decomposition of the diagonal, in the first lecture. The decomposition of the diagonal controls many other stable birational invariants and I will focus in the second lecture on invariants related to the Abel-Jacobi map and the coniveau.

On Rational Cubic Fourfolds Containing Veronese Surfaces
Yu Wei Fan

Veronese surfaces in P^5 induce birational automorphisms on P^5. We show that these birational automorphisms give rise to a non-trivial birational involution on the space of cubic fourfolds containing a Veronese surface. Using this involution, we prove the rationality of certain cubic fourfolds that was not known previously. We also prove that if a cubic fourfold X contains a Veronese surface, then the only non-isomorphic cubic that shares the same K3 category with X is the image of X under the Cremona transformation. Joint work with Kuan-Wen Lai.