Tropical Geometry and Sandpiles

An IMSA program
17-22 November 2019
Coral Gables, FL

https://www.imsa.miami.edu/events/fall-2019-emphasis-semester/index.html

PURPOSE

In the late 80's, motivated by the study of char we call now complex systems, physicists stumbled upon a deceptively simple cellular automaton as a toy model for self-organized criticality and stated theorems that from the mathematical perspective are remarkable conjectures. In the last few years a spurt of activity surrounding these conjectures has happened, in particular new connections to the XX! Century field of Tropical Geometry were found. These connections suggest two new avenues of research, in one direction with the advent of quantum toric geometry and quantum tropical geometry, the theory of sandpiles is insensitive to the quantization and provides a guide as to this geometric realm which suggest an amplification to the homological mirror symmetry program, in the opposite direction homological mirror symmetry may provide inspiration to deal with some of the conjectures regarding self-organized criticality in sandpiles.

PROGRAM

Sunday: Introductory seminar, 7pm Lobby of the hotel.

	Mon 18 Nov	Tue	Wed	Thu	Fri
9 am -10 am	Lupercio *10	Kerr	Kerr	Otero	Verjovsky
10:30 am - 11:30 am	Lang *11:30	López de Medrano	Katzarkov	Lupercio	Lupercio
2:30 pm - 3:30 pm	Kalinin	Kalinin	Free afternoon	Uribe	Cruz
4pm - 5pm	Gendron	Ruiz-Guido	Free afternoon	Angel	Open problems round table

MINI-COURSES

1. *Elements of Mirror Symmetry* by Gabriel Kerr
2. *Sandpiles and Tropical Geometry* by Nikita Kalinin
3. *Sandpiles and Quantum Toric Geometry and Chimeras* by Ernesto Lupercio

SPEAKERS

Andres Ángel (U Norte, Barranquilla)

Pablo Cruz (Cinvestav, México)

Tim Gedron (UNAM, Cuernavaca)

Moritz Lang (IST, Vienna)

Lucía López de Medrano (UNAM, Cuernavaca)

Ernesto Lupercio (Cinvestav, México)

Nikita Kalinin (HSE, St Petersburg)

Ludmil Katzarkov (U Miami)

Gabriel Kerr (Kansas State)

Carlos Ruiz-Guido

Ignacio Otero (Cinvestav)

Bernardo Uribe (U Norte, Barranquilla)

Alberto Verjovsky (UNAM, Cuernavaca)

TITLES ABSTRACTS

Andres Ángel (U Norte, Barranquilla)

Pablo Cruz (Cinvestav, México)

Experiments in Cellular automata and Entropy.

Tim Gedron (UNAM, Cuernavaca)

Quasicrystal Drinfeld Modules and the Fine Structure of Quantum Tori

The quantum modular invariant is a multi-valued, discontinuous function on the moduli space of quantum tori,

\[j^{\rm qt}:\R/{\rm GL}_{2}\Z\multimap \R \cup \{ \infty\},\]

defined using diophantine approximations.

For a quadratic real number θ, the Main Conjecture is (roughly) that the product of the multi-values of $j^{\rm qt}(\uptheta)$ generate the Hilbert class field of $K=\Q (\uptheta)$.

The analog of this conjecture in the function field setting was proved in 2016 by Demangos-Gendron. There, for f quadratic, $j^{\rm qt}(f)$ has finitely many values, each of which is the modular invariant of a 1-dimensional Drinfeld module. The conjecture is proved using elements of Drinfeld-Hayes theory: an explicit class field theory which uses ``small rings of integers" that are defined geometrically using the curve associated to f.

In 2017, Pink showed that in the number field setting, for θ quadratic, the values of $j^{\rm qt}(\theta)$ are given by the modular invariants of certain 1-dimensional quasicrystals. In this talk, we show how these quasicrystals may be used to define the analog of Drinfeld modules in characteristic zero. There is a conjectural Drinfeld-Hayes theory contemplated, and if achieved, it is expected to yield a proof of the Main Conjecture

Moritz Lang (IST, Vienna)

Sandpile Dynamics

Lucía López de Medrano (UNAM, Cuernavaca)

Tropical Topology

Ernesto Lupercio (Cinvestav, México)

Sandpiles, Stacks, Quantum Gravity and Chimeras

Nikita Kalinin (HSE, St Petersburg)

Tropical Sandpiles

Ludmil Katzarkov (U Miami)

TBA

Gabriel Kerr (Kansas State)

Title : Tropical geometry, Mori dream spaces and mirror symmetry

Abstract : In this talk I will discuss a result which completely describes the GIT fan of a Mori dream space and identifies it as the dual of a polytope called the μ-secondary polytope $\Sigma_\mu (A)$. This polytope is defined using the data of a finite set of characters $A \subset Hom((\mathbb{C}^*)^m, \mathbb{C}^*) = \Lambda$, and a measure μ on Λ. In the case of a Mori dream space, μ is the volume measure of the tropicalization of the spectrum of the Cox ring. I will explain how this result opens the door to understanding mirror decompositions of the categories in homological mirror symmetry. This is a joint work with Jamie Peabody.

Title : A tropical approach to homological mirror symmetry of quadrics

Abstract : There have been many descriptions of the mirror potential to a quadric, starting with Hori and Vafa and continuing through Abouzaid-Auroux-Katzarkov and Pech-Rietsch-Williams. I will describe a new approach using a different anti-canonical divisor and tropical geometry. In all cases, there is not a proof of the homological mirror symmetry conjecture equating the derived category of sheaves on the quadric with a Fukaya-Seidel category on the potential. Nevertheless, an exceptional collection of sheaves was identified in the work of Kapranov and the potential we construct has a natural exceptional collection which is likely mirror to Kapranov's. I will explain this observation for the two dimensional quadric (where neither side of the conjecture looks similar to the toric case). This is a joint work with Reginald Anderson and Yijia Liu.

Carlos Ruiz-Guido

AModel Theorectic Künneth Formula

One of the main upshots from modern model theory is the concept of stability. When trying to generalize these ideas to more general theories Hrushovski isolated an important property called amalgamation of types. I will present some of its consequences and relations to known results.

Ignacio Otero (Cinvestav)

Chimeric Geometry

Bernardo Uribe (U Norte, Barranquilla)

Fiberwise Pontrjagin duality in multiplicative gerbes

Abstract: In this talk I will outline a duality that can be associated to multiplicative gerbes. This duality in particular implies that the categories of representations in gerbes of the fiberwise multiplicative gerbas are equivalent.

Alberto Verjovsky (UNAM, Cuernavaca)

Toric proalgebraic laminations

Alberto Verjovsky

Abstract

This is Joint work with Juan Manuel Burgos. We present several results about toric proalgebraic laminations. These "solenoidal laminations" are obtained as inverse limits of branched self-coverings of toric manifods. A paradigmatic example is the space $\hat{\mathbb{C}}_{\mathbb{Q}}$ obtained as the inverse limit of the maps of the Riemann sphere $\hat{\mathbb{C}}_{\mathbb{Q}} = P^1_{\mathbb{C}}$ given by $z \mapsto z^n$ $(n \in \mathbb{N})$. Then $\hat{\mathbb{C}}_{\mathbb{Q}}$ is homeomorphic to the topological suspension of the one-dimensional solenoid $S_{\mathbb{Q}}$ which is the Pontryagin dual of the additive rationals $\mathbb{Q}$ with its discrete topology. Outside the two suspension points (which correspond to $0, \infty \in \hat{\mathbb{C}}$) the space is a Riemann surface lamination with leaves densely immersed copies of $\mathbb{C}^*$. In fact this set is the profinite completion $\mathbb{C}^*_{\mathbb{Q}}$ of the algebraic torus $\mathbb{C}^*$. We call this singular lamination *the adèlic projective line*. In [1, 2, 3] we study the properties of the adèlic projective line. We present results that show that this type of construction applies to any projective toric variety (or toric orbifold).

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